Consequences, opportunities, and Arrovian theorems with consequentialist domains

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Abstract

We examine the possibility of Arrovian social choice when alternatives are pairs of outcomes and opportunity sets from which they are chosen. Consequentialism is a choice attitude towards outcomes and opportunities and prioritizes outcomes rather than opportunities for choice. We first provide a sufficient condition for a restricted domain on which Arrow's impossibility theorem holds. Then it is shown that a domain such that all individuals are the same type of consequentialist satisfies the proposed domain condition. On the other hand, if there exist different types of consequentialist in society, it is possible to construct an *extended social welfare function* (ESWF) which satisfies *weak Pareto*, *independence of irrelevant alternatives* and *nondictatorship*. However, if we additionally impose *Pareto indifference*, *anonymity* or *neutrality* on an ESWF, then Arrow's impossibility theorem holds even if there simultaneously exist different types of consequentialist in society. This statement amends a theorem in Suzumura and Xu (2004).

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1 Introduction

This paper extends the informational basis of Arrovian social choice theory and explores the possibility of a desirable collective decision-making. Arrovian social choice theory has been mainly premised on individuals' welfare enjoyed from their consequences in assessing a social state and an economic policy and paid little attention to nonwelfaristc features of consequences and nonconsequential features such as decision-making procedures and opportunities for choice in social welfare analysis.

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In this paper, however, we take into account the intrinsic value of opportunities for choice, since choosing itself can be seen as a valuable functioning. To capture the intrinsic value of opportunities for choice we allow all individuals to express the following type of preference: Choosing an outcome x from an opportunity set A is better than choosing another outcome y from an opportunity set B. Such an *extended preference ordering* is useful in considering the following situations.[1]

Consider a location problem of a military base in a country. Suppose that there exist two possible sites, the *North* area and the *South* area. Then consider the following situation:

$$\{(s, \{s\}), (s, \{n, s\})\},$$

where n stands for the North area and s stands for the South area. In this situation individuals in the South area may prefer $(s, \{n, s\})$ to $(s, \{s\})$ because in $(s, \{s\})$ they are forced to locate the military base, on the other hand in $(s, \{n, s\})$ the military base is finally located in the South area but the North area is considered as an alternative site, and then they express the intrinsic value of opportunities for choice. Extended preference orderings can grasp such a comprehension of welfare evaluation.

For another example, consider that individuals face a choice problem of a political party, and have two one-party rules, one is dictatorship by the Left and the other is dictatorship by the Right. Formally, they are defined as

$$\{(l, \{l\}), (r, \{r\})\},$$

where l stands for the Left and r stands for the Right. We assume that an individual chooses the Left over the Right, that is, $(l, \{l\})$ is preferred to $(r, \{r\})$. Now, suppose that the political system transforms from dictatorship into democracy and the Center (c) forms against the Right. Then consider the following choice situation:

$$\{(l, \{l\}), (r, \{r, c\})\}.$$

In this situation he may choose $(r, \{r, c\})$ over $(l, \{l\})$ since he prefers democracy to dictatorship, and then he compensates the poorness of consequences by the richness of opportunities. Thus, extended preference orderings can express such a substisution of welfare evaluation.

When alternatives are pairs of outcomes and opportunity sets from which they are chosen, the problem of social choice is to make a collective decision based on individuals' extended preference orderings. In a recent work Suzumura and Xu (2004) discuss Arrow's impossibility

[1] See Iwata (2006) and Suzumura and Xu (2001,2003,2004) for the discussion of extended preference orderings.

theorem (Arrow 1963) in this extended framework. They consider two frameworks, the *consequentialist framework* and the *nonconsequentialist framework*, and explore resolutions to Arrow's impossibility theorem.

The implication of their analysis is that whereas in the consequentialist framework, the diversity of choice attitudes towards outcomes and opportunities plays an important role in resolving Arrow's impossibility theorem, in the nonconsequentialist framework, it is possible to guarantee the existence of a resolution of Arrow's impossibility theorem as long as there exists at least one nonconsequentialist in society, and this result is not nullified even in the homogeneous society where there exists a similarity of choice attitudes towards outcomes and opportunities.

In this paper we investigate the consequentialist framework. Consequentialism is a choice attitude towards outcomes and opportunities for choice. It prioritizes outcomes rather than opportunities, but it does not always ignore the richness of opportunities. We are espetially interested in investigating to what extent we can obtain a possible solution to Arrow's impossibility theorem in the context of consequentialism. That is, we examine whether we can construct a reasonable *extended social welfare function* (ESWF) within consequentialism.

Technically, consequentialists are defined by restricting the set of their admissible extended preference orderings. We assume that there exist two types of consequentialist, *extreme consequentialists* and *strong consequentialists*, in society and for an individual who is not a consequentialist, his preferences are not restricted at all.

We first provide a class of domains on which Arrow's impossibility theorem holds. This class has a similar structural property to a preference domain discussed in Arrovian social choice thoery for economic environments, where individuals' preferences are supposed to be continuous, convex and strictly monotonic.[2] If all individuals are either extreme consequentialists or strong consequentialists, then the corresponding domains are included in the proposed class of domains. Moreover, we show that it is possible to construct an ESWF which satisfies *weak Pareto principle*, *independence of irrelevant alternatives* and *nondictatorship* under the diversity of choice attitudes within consequentialism. That is, if there simultaneously exist at least one extreme consequentialist and at least one strong consequentialist in society, then we can dissolve Arrow's impossibility theorem.

We also impose *Pareto indifference*, *anonymity* or *neutrality* on an ESWF. Suzumura and Xu's (2004) analysis is the same as ours if we additionally require that an ESWF satisfies Pareto indifference. It will be shown that we get impossibility results in Suzumura and Xu (2004)

[2] Arrovian social choice in economic environments have studied by many authors. See, for example, Kalai et al. (1979), Bordes and Le Breton (1989), Bordes and Le Breton (1990). See also Le Breton and Weymark (2005) for the survey of the literature of Arrovian social choice in economic environments.

from our results. We also show that it is impossible to design an ESWF which satisfies Arrow's conditions and one of the above additional conditions even if there exists the diversity of choice attitudes within consequentialism. This statement is incompatible with a theorem in Suzumura and Xu (2004) and in fact amends it.

Section 2 presents basic notation. Section 3 introduces a domain condition within the extended framework. In Section 4, we examine Arrow's impossibility theorem within consequentialism. In Section 5, we additionally impose Pareto indifference on ESWF and compare our results with results in Suzumura and Xu (2004), and in section 6 we require that ESWF additionally satisfies anonymity or neutrality and get impossibility results. Section 7 concludes the discussion.

2 Notation

Let X be the set of all social states; we assume that $3 \leq \sharp X = m < \infty$. The elements of X are denoted by $x, y, z, \ldots$, and they are called *outcomes*. Let K be the set of all non-empty subsets of X. The elements of K are denoted by $A, B, C, \ldots$, and they are called *opportunity sets*. $X \times K$ denotes the Cartesian product of X and K. The elements of $X \times K$ are denoted by $(x, A), (y, B), (z, C), \ldots$, and they are called *extended alternatives*. Let $\Omega = \{(x, A) \in X \times K | A \in K \text{ and } x \in A\}$. That is, Ω contains all (x, A) such that $x \in A$ whenever $(x, A) \in \Omega$. The interpretation of $(x, A) \in \Omega$ is that the outcome x is chosen from the opportunity set A.

Let $N = \{1, \ldots, n\}$, $2 \leq \sharp N < \infty$, be the set of all individuals in society. For each $i \in N$, R_i is an *extended preference ordering* over Ω, which is reflexive, transitive and complete. The asymmetric and symmetric parts of R_i are denoted by P_i and I_i respectively. For any $(x, A), (y, B) \in \Omega$, $(x, A) R_i (y, B)$ implies that choosing x from A is for i at least as good as choosing y from B.

Let $\mathcal{R}(\Omega)$ be the set of all logically possible orderings over Ω. $\mathcal{L}(\Omega)$ is the set of all linear orderings over Ω. A domain $\mathcal{D}$ is a subset of $\mathcal{R}(\Omega)^n$; an element of $\mathcal{D}$ is a *profile* and it is denoted by $\mathbf{R} = (R_1, \ldots, R_n)$. An *extended social welfare function* (ESWF) is a function $f : \mathcal{D} \to \mathcal{R}(\Omega)$. $R = f(\mathbf{R})$ means a social preference relation. The social strict preference relation and the social indifference relation are P and I respectively.

Let $\mathcal{D}_i \subseteq \mathcal{R}(\Omega)$ be an *admissible individual preference domain for* $i \in N$. A domain $\mathcal{D}$ is a *Cartesian preference domain* if and only if there exist $\mathcal{D}_1, \ldots, \mathcal{D}_n \subset \mathcal{R}(\Omega)$ such that $\mathcal{D} = \mathcal{D}_1 \times \cdots \times \mathcal{D}_n$. A domain $\mathcal{D}$ is a *common domain* if, for all $i, j \in N$, $\mathcal{D}_i = \mathcal{D}_j$. $\mathcal{D} = \mathcal{R}(\Omega)^n$ is called the *universal domain*. Any $\mathcal{D} \subsetneq \mathcal{R}(\Omega)^n$ is said to be *restricted*.

For all subset Γ of Ω, $\mathcal{R}(\Omega)|_\Gamma$ is the restriction of $\mathcal{R}(\Omega)$ to Γ. Let $\mathcal{D}|_\Gamma$ be the restriction of $\mathcal{D}$

to Γ. If $\sharp(\mathcal{D}_i|_\Gamma) = 1$, then $\Gamma \subset \Omega$ is *trivial with respect to* $\mathcal{D}_i$. If there exists $i \in N$ such that Γ is trivial with respect to $\mathcal{D}_i$, then Γ is *trivial with respect to* $\mathcal{D}$. If Γ is not trivial with respect to $\mathcal{D}_i$, then Γ is said to be *nontrivial with respect to* $\mathcal{D}_i$. Note that Γ is nontrivial with respect to $\mathcal{D}$ if it is nontrivial with respect to $\mathcal{D}_i$ for all $i \in N$. If $\mathcal{D}_i = \mathcal{R}(\Omega)|_\Gamma$, then Γ is *free with respect to* $\mathcal{D}_i$. If, for all $i \in N$, Γ is free with respect to $\mathcal{D}_i$, then Γ is *free with respect to* $\mathcal{D}$. If Γ is free (with respect to $\mathcal{D}$), we may also say that $\mathcal{D}$ is *locally unversal* on Γ. If, for $\sharp\Gamma = 3$, Γ is free with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$), then Γ is a *free triple with respect to* $\mathcal{D}_i$ (resp. $\mathcal{D}$).

3 Domain restrictions within the extended framework

We introduce Arrow's well-known conditions on an ESWF.

Weak Pareto (WP). For all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{D}$, if $(x, A)P_i(y, B)$ holds for all $i \in N$, then $(x, A)P(y, B)$, where $R = f(\mathbf{R})$.

Independence of irrelevant alternatives (IIA). For all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R}^1 = (R_1^1, \ldots, R_n^1)$, $\mathbf{R}^2 = (R_1^2, \ldots, R_n^2) \in \mathcal{D}$, if $\mathbf{R}^1$ and $\mathbf{R}^2$ coincide on $\{(x, A), (y, B)\}$, then R^1 and R^2 coincide on $\{(x, A), (y, B)\}$, where $R^1 = f(\mathbf{R}^1)$ and $R^2 = f(\mathbf{R}^2)$.

Nondictatorship (ND). There is no individual $i \in N$ such that for all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{D}$, if $(x, A)P_i(y, B)$, then $(x, A)P(y, B)$, where $R = f(\mathbf{R})$.

We now provide a domain condition in this paper, which has a similar structural property to domain restrictions discussed in Arrovian social choice thoery on economic domains. In the next section, we will show that some consequentialist domains satisfy the proposed domain condition. Let an *intermediate domain* (I-domain) be a Cartesian domain $\mathcal{D}$ such that $\mathcal{L}(\Omega) \subset \mathcal{D}_i \subset \mathcal{R}(\Omega)$ for all $i \in N$. Since I-domains when $\sharp\Omega = 3$ play a crucial role in what follows, we consider a situation such that Ω is a triple, that is $\Omega = \{(x, A), (y, B), (z, C)\}$.[3] A pair $\{(u, D), (v, E)\}$ is *distinct* if $(u, D) \neq (v, E)$. An I-domain is an *Arrow inconsistent domain* (A-domain) if:

For all two distinct pairs of elements in Ω, $\{(u, D), (v, E)\}$ and $\{(u, D), (w, F)\}$, for all $i \in N$, if R_i^* such that $(u, D)I_i^*(v, E)I_i^*(w, F)$ belongs to $\mathcal{D}_i$, then there exist at least one of R_i^1 or R_i^2, and at least one of R_i^3 or R_i^4 such that $(w, F)P_i^1(u, D)I_i^1(v, E)$, $(u, D)I_i^2(v, E)P_i^2(w, F)$, $(v, E)P_i^3(u, D)I_i^3(w, F)$ and $(u, D)I_i^4(w, F)P_i^4(v, E)$.

It is easy to cheak that if Ω is a free triple with respect to $\mathcal{D}$, then $\mathcal{D}$ is an A-domain. The following lemma is useful to prove our results in this paper.

[3]For the case where alternatives contain more than three elements, Kelly (1994) discusses Arrow's impossibility theorem on I-domain.

Lemma 1 For $\sharp\Omega = 3$, there exists no extended social welfare function on any A-domain which satisfies (WP), (IIA) and (ND).

Proof. By an analogical argument to Bordes and Le Breton (1990, Thoerem 3). ■

Before we define the domain condition, we introduce a few definitions. We begin defining the *conditional* (on X) *relative to outcomes* (or CRO) subsets of Ω as $\Omega_X := \{(x, X) \in \Omega \mid x \in X\}$. For all $i \in N$, a pair $\{(x, A), (y, B)\} \subset \Omega$ is said to be *CRO trivial with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$)* if and only if the pair $\{(x, X), (y, X)\} \subset \Omega_X$ is trivial with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$). For all $i \in N$, a pair $\{(x, A), (y, B)\} \subset \Omega$ is said to be *CRO nontrivial with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$)* if and only if the pair $\{(x, X), (y, X)\} \subset \Omega_X$ is nontrivial with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$). A subset $\{(x_1, A_1), (x_2, A_2), \ldots, (x_q, A_q)\} \subset \Omega$ is said to be *CRO free with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$)* if and only if $\{(x_1, X), (x_2, X), \ldots, (x_q, X)\} \subset \Omega_X$ is free with respect to $\mathcal{D}_i$ (resp. $\mathcal{D}$).

We now define what we call *extendedly saturating domains*, which corresponds to domain conditions discussed in Arrovian social choice for economic environments in which saturating domains (Kalai et al., 1979) and ultrasaturating domains (Bordes and Le Breton, 1990) are defined respectively for the case of public alternatives only and the mixed case of public and private alternatives.

Definition 1 A domain $\mathcal{D}$ is *extendedly saturating* if and only if it satisfies all of the following conditions:

1. If a triple $\{(x, A), (y, B), (z, C)\} \subset \Omega$ is CRO free with respect to $\mathcal{D}$, then for all $A^1, A^2, A^3 \in K$ such that $x \in A^1, y \in A^2, z \in A^3$, the restriction of $\mathcal{D}$ to the triple $\{(x, A^1), (y, A^2), (z, A^3)\}$ is an A-domain.

2. For all pairs $\{(x, A), (y, B)\} \subset \Omega$ and $\{(z, C), (w, D)\} \subset \Omega$ which are CRO nontrivial with respect to $\mathcal{D}$, there exist $(x^1, A^1), (x^2, A^2), \ldots, (x^T, A^T) \in \Omega$ such that:

$$(x^1, A^1) = (x, A), (x^2, A^2) = (y, B), (x^{T-1}, A^{T-1}) = (z, C), (x^T, A^T) = (w, D);$$

   and:

   for all t from 1 to $T - 2$:

$$\Gamma^t = \{(x^t, A^t), (x^{t+1}, A^{t+1}), (x^{t+2}, A^{t+2})\}$$

   is a CRO free triple with respect to $\mathcal{D}$.

3. There exist in Ω at least two distinct pairs which are CRO nontrivial with respect to $\mathcal{D}$.

We refer to some comments for the above three conditions. Extended alternatives are pairs of outcomes and opportunity sets from which they are chosen and contain no private components. In this sense extendedly saturating domains are extensions to saturating domains. On the other hand, each condition defining extendedly saturating domains is similar to a condition defining ultrasaturating domains.

Conditions (1), (2) and (3) of extendedly saturating domains respectively are similar to conditions (3), (4) and (7) of ultrasaturating domains. Since condition (5) of ultrasaturating domains is a particular condition for the cases where there are private components (see, for example, Bordes and Le Breton, 1989), it is not required for the definition of extendely saturating domains. We will discuss a similar condition to condition (6) of ultrasaturating domains in the subsequent section.

This paper considers the additional assumption, which is similar to condition (1) of ultrasaturating domains.

Domain Assumption: For all pairs $\{(x, A), (y, B)\} \subset \Omega$, the pair is CRO nontrivial with respect to $\mathcal{D}$ if and only if it is nontrivial with respect to $\mathcal{D}$.

Lastly, we conclude this section by proving the following result.

Theorem 1 Suppose a common and extendedly saturating domain $\mathcal{D}$ satisfies Domain Assumption. Then, there exists no extended social welfare function defined on $\mathcal{D}$ which satisfies (WP), (IIA) and (ND).

4 Arrow's theorem within the consequentialist framework

In this section we examine whether Arrow's impossibility theorem holds within cosequentialism. Let us now identify two versions of consequentialist.

Extreme consequentialist. An individual $i \in N$ is said to be an *extreme consequentialist* if, for all $(x, A), (x, B) \in \Omega$, $(x, A) I_i (x, B)$.

Strong consequentialist. An individual $i \in N$ is said to be a *strong consequentialist* if, for all $(x, A), (y, B) \in \Omega$, if $(x, \{x\}) I_i (y, \{y\})$, then $\sharp A \geq \sharp B \Leftrightarrow (x, A) R_i (y, B)$; and if $(x, \{x\}) P_i (y, \{y\})$, then $(x, A) P_i (y, B)$.

Thus, an extreme consequentialist judges exclusively two extended alternatives (x, A) and (y, B) on their outcomes x and y, and the opportunity sets A and B from which x and y are chosen are irrelevant. On the other hand, a strong consequentialist ranks two extended alternatives (x, A) and (y, B) as follows: when the individual has a strict preference over $(x, \{x\})$

and $(y, \{y\})$, the ranknig between (x, A) and (y, B) corresponds to that of $(x, \{x\})$ and $(y, \{y\})$. If he is indifferent between $(x, \{x\})$ and $(y, \{y\})$, he ranks (x, A) and (y, B) in terms of the cardinality of elements in opportunity sets A and B.

From now on, we introduce three domains on an ESWF by specifying individuals in society. Let $\mathcal{D}_e$ be the set of admissible extended preference orderings of an extreme consequentialist. Let $\mathcal{D}_s$ be the set of admissible extended preference orderings of a strong consequentialist. The first domain is $\mathcal{D}_E = \mathcal{D}_e \times \cdots \times \mathcal{D}_e$. That is, $\mathcal{D}_E$ is a domain such that all individuals are extreme consequentialists. In the second place, let $\mathcal{D}_S = \mathcal{D}_s \times \cdots \times \mathcal{D}_s$. That is, $\mathcal{D}_S$ is a domain such that all individuals are strong consequentialists. Finally, we introduce $\mathcal{D}_{ES} = \mathcal{D}_e \times \mathcal{D}_s \times \mathcal{R}(\Omega)^{n-2}$. The domain $\mathcal{D}_{ES}$ is defined in such a way that there exsit one extreme consequentialist and one strong consequentialist in society and any other people's preferences are not restricted at all.

We first prove the following two propositions.

Proposition 1 The domains $\mathcal{D}_E$, $\mathcal{D}_S$ and $\mathcal{D}_{ES}$ satisfy Domain Assumption.

Proposition 2 The domains $\mathcal{D}_E$ and $\mathcal{D}_S$ are common and extendedly saturating.

From Theorem 1 and Propositions 1 and 2, we have the following results.

Theorem 2 There exists no extended social welfare function defined on $\mathcal{D}_E$ which satisfies (WP), (IIA) and (ND).

Theorem 3 There exists no extended social welfare function defined on $\mathcal{D}_S$ which satisfies (WP), (IIA) and (ND).

The facts that a domain is common and extendedly saturating and satisfies Domain Assumption is merely sufficient but not necessary, for a domain to hold Arrow's impossibility theorem. Moreover, that a domain is not common is essential for a resolution to Arrow's impossibility theorem within consequentialism. By Proposition 1, the domain $\mathcal{D}_{ES}$ satisfies Domain Assumption. It is easy to confirm that the domain $\mathcal{D}_{ES}$ is extendedly saturating, but it is not clearly common.[4] Therefore, we can not apply Theorem 1 to the domain $\mathcal{D}_{ES}$, and in fact, it is possible to construct an ESWF which satisfies (WP), (IIA) and (ND).

Theorem 4 There exists an extended social welfare function defined on $\mathcal{D}_{ES}$ which satisfies (WP), (IIA) and (ND).

[4]That the domain $\mathcal{D}_{ES}$ is extendedly saturating follows from an analogical reasoning of Proposition 2. See also the proof of Proposition 3.

We follow with remarks. Suzumura and Xu (2004) consider the following domain condition. Let $\mathcal{D}(E \cup S)$ be a class of domains such that there exist at least one individual is an extreme consequentialist and at least one individual is a strong consequentialist. The domain $\mathcal{D}_{ES}$ is the largest domain in $\mathcal{D}(E \cup S)$. If there exist just two individuals in society, then $\mathcal{D}(E \cup S)$ is equivalent to $\mathcal{D}_{ES}$. By the proposed ESWF in Theorem 4 we can prove that there exists an ESWF defined on $\mathcal{D}(E \cup S)$ which satisfies (WP), (IIA) and (ND).

5 Pareto indifference

In this section we imopse an additional condition, i.e. *Pareto indiffernce* on an ESWF, as in Suzumura and Xu (2004).[5]

Pareto indifference (PI). For all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{D}$, if $(x, A)I_i(y, B)$ holds for all $i \in N$, then we have $(x, A)I(y, B)$, where $R = f(\mathbf{R})$.

(PI) requires that if all individuals are indifferent between (x, A) and (y, B), the two extended alternatives are socially indifferent. Then we get the following results from Theorems 2 and 3.

Corollary 1 (Suzumura and Xu 2004, Theorem 1) There exists no extended social welfare function on $\mathcal{D}_E$ which satisfies (WP), (PI), (IIA) and (ND).

Corollary 2 (Suzumura and Xu 2004, Theorem 3) There exists no extended social welfare function on $\mathcal{D}_S$ which satisfies (WP), (PI), (IIA) and (ND).

On the other hand, Suzumura and Xu (2004, Theorem 2) say that it is possible to construct an ESWF defined on $\mathcal{D}(E \cup S)$ which satisfies (WP), (PI), (IIA) and (ND). However, we will amend this statement.

The following ESWF is constructed by Suzumura and Xu (2004) to prove their Theorem 2. Suppose that $e \in N$ is an extream consequentialist and $s \in N$ is a strong consequentialist. For all $(x, A), (y, B) \in \Omega$, if $(x, \{x\})P_s(y, \{y\})$, then $(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)$; if $(x, \{x\})I_s(y, \{y\})$, then $(x, A)R(y, B) \Leftrightarrow (x, A)R_e(y, B)$, where $R = f(\mathbf{R})$.

The next example shows that this ESWF does not satisfy (IIA).

Example 1 Consider the ESWF defined above, and two extended alternatives $(x, A), (y, B) \in \Omega$ and any two profiles $\mathbf{R}^1$,$\mathbf{R}^2 \in \mathcal{D}(E \cup S)$ such that for all $i \in N \backslash \{e, s\}$, $(x, A)R_i^1(y, B) \Leftrightarrow (x, A)R_i^2(y, B)$, $(x, A)I_e^1(y, B)$, $(x, A)I_e^2(y, B)$, $(x, A)P_s^1(y, B)$, $(x, A)P_s^2(y, B)$, $(x, \{x\})P_s^1(y, \{y\})$

[5] Suzumura and Xu (2004) combine Pareto indifference with weak Pareto and call it strong Pareto.

and $(x, \{x\}) I_s^2 (y, \{y\})$. It is easy to confirm that if $x \neq y$ and $\sharp A > \sharp B$, there exist profiles such as $\mathbf{R}^1$ and $\mathbf{R}^2$ in $\mathcal{D}(E \cup S)$.

It is clear that for all $i \in N$ $(x, A) R_i^1 (y, B) \Leftrightarrow (x, A) R_i^2 (y, B)$. However, by the proposed ESWF, we have $(x, A) P^1 (y, B)$ and $(x, A) I^2 (y, B)$, where $R^1 = f(\mathbf{R}^1)$ and $R^2 = f(\mathbf{R}^2)$. Therefore, this ESWF does not satisfy (IIA).

If we can show that there exists no ESWF defined on $\mathcal{D}_{ES}$ which satisfies (WP), (PI), (IIA) and (ND), then this statement is incompatible with Suzumura and Xu's (2004) Theorem 2. Therefore, our goal in this section is to prove that there exsits no ESWF on $\mathcal{D}_{ES}$ which satisfies (WP), (PI), (IIA) and (ND).

To show this result, we introduce an additional domain condition, which has a similar property to condition (6) of ultrasaturating domains. If a CRO trivial pair $\{(x, A), (y, B)\}$ with respect to $\mathcal{D}_i$ is *separable with respect to* $\mathcal{D}_i$, there exist an extended alternative (z, C) such that $\{(x, A), (z, C)\}$ and $\{(y, B), (z, C)\}$ are CRO nontrivial with respect to $\mathcal{D}_i$ and either (a) $(x, A) P_i (y, B)$ for all $R_i \in \mathcal{D}_i$ and there exists $R_i' \in \mathcal{D}_i$ such that $(x, A) P_i' (z, C) P_i' (y, B)$ or (b) $(y, B) P_i (x, A)$ for all $R_i \in \mathcal{D}_i$ and there exists $R_i' \in \mathcal{D}_i$ such that $(y, B) P_i' (z, C) P_i' (x, A)$. The next example shows that there exists a pair of extended alternatives which is not separable with respect to $\mathcal{D}_s$.

Example 2 Let $\{(x, A), (y, B)\}$ be such that $x = y$ and $\sharp A - \sharp B = 1$. Then $\{(x, A), (y, B)\}$ is clearly a CRO trivial pair with respect to $\mathcal{D}_s$. Then $\{(x, A), (z, C)\}$ and $\{(y, B), (z, C)\}$ are CRO nontrivial with respect to $\mathcal{D}_s$ if and only if $x = y \neq z$. However, we have $(x, A) P_s (y, B)$ for all $R_s \in \mathcal{D}_s$, but there exists no $R_s' \in \mathcal{D}_s$ such that $(x, A) P_s' (z, C) P_s' (y, B)$.

For this reason, we do not use this separation condition. Instead, we adopt a modified condition. It requires that for a CRO trivial pair with respect to $\mathcal{D}$, there exists an extended alternative which satisfies a modified separation condition for all $i \in N$.

Definition 2 A CRO trivial pair $\{(x, A), (y, B)\} \subset \Omega$ with respect to $\mathcal{D}$ is *less separable with respect to* $\mathcal{D}$ if there exists an extended alternative (z, C) such that:
for all $i \in N$, $\{(x, A), (z, C)\}$ and $\{(z, C), (y, B)\}$ are CRO nontrivial with respect to $\mathcal{D}_i$, and for all $R_i \in \mathcal{D}_i$ one of the following conditions holds: (a) if $(x, A) P_i (y, B)$, then there exists $R_i' \in \mathcal{D}_i$ such that $(x, A) I_i' (z, C) P_i' (y, B)$; (b) if $(y, B) P_i (x, A)$, then there exists $R_i' \in \mathcal{D}_i$ such that $(y, B) P_i' (z, C) I_i' (x, A)$; and (c) if $(x, A) I_i (y, B)$, then there exists $R_i' \in \mathcal{D}_i$ such that $(x, A) I_i' (z, C) I_i' (y, B)$.

If a pair $\{(x, A), (y, B)\}$ is CRO trivial with respect to $\mathcal{D}$, then there exist an individual $i \in N$ such that it is CRO trivial with respect to $\mathcal{D}_i$, that is, $\{(x, X), (y, X)\}$ is trivial with respect to $\mathcal{D}_i$. Note that this separation condition is required for all individuals, and for some $i \in N$ a pair $\{(x, A), (y, B)\}$ may be CRO nontrivial with respect to $\mathcal{D}_i$, even if it is CRO trivial with respect to $\mathcal{D}$. We use this modified separation condition to define *extendedly strongly saturating domains*.

Definition 3 A domain $\mathcal{D}$ is extendedly strongly saturating if and only if (a) $\mathcal{D}$ is extendedly saturating and (b) all CRO trivial paris $\{(x, A), (y, B)\} \subset \Omega$ with respect to $\mathcal{D}$ are less separable with respect to $\mathcal{D}$.

With an extendedly strongly saturating domain, all CRO trivial pairs with respect to $\mathcal{D}$ are less separable with respect to $\mathcal{D}$. We show that there exists no ESWF defined on an extendedly strongly saturating domain which satisfies (WP), (PI), (IIA) and (ND).

Theorem 5 There exists no extended social welfare function defined on an extendedly strongly saturating domain which satisfies (WP), (PI), (IIA) and (ND).

The next result shows that the domain $\mathcal{D}_{ES}$ is extendedly strongly saturating.

Proposition 3 The domain $\mathcal{D}_{ES}$ is extendedly strongly saturating.

Finally our result follows from Theorem 5 and Proposition 3.

Theorem 6 There exists no extended social welfare function defined on $\mathcal{D}_{ES}$ which satisfies (WP), (PI), (IIA) and (ND).

6 Anonymity and Neutrality

In the previous section we constructed an ESWF defined on $\mathcal{D}_{ES}$ which satisfies (WP), (IIA) and (ND). However, the proposed ESWF does not satisfy other desirable conditions like *anonymity* and *neutrality*. In this section we examine whether it is possible to construct an anonymous or neutral ESWF defined on $\mathcal{D}_{ES}$ which satisfies Arrow's conditions. First we define anonymity and neutrality. Let a permutation π be a bijection from N to itself. Given π and $\mathbf{R} \in \mathcal{D}$, let $\pi(\mathbf{R}) = (R_{\pi(1)}, \ldots, R_{\pi(n)})$.

Anonymity (A). For all $(x, A), (y, B) \in \Omega$ and for all $\mathbf{R}^1 \in \mathcal{D}$, if there exists a profile $\mathbf{R}^2 \in \mathcal{D}$ such that $\mathbf{R}^2 = \pi(\mathbf{R}^1)$ for a permutation π, then $(x, A)R^1(y, B) \Leftrightarrow (x, A)R^2(y, B)$, where $R^1 = f(\mathbf{R}^1)$ and $R^2 = f(\mathbf{R}^2)$.

Neutrality (N). For all $(x, A), (y, B), (z, C), (w, D) \in \Omega$ and for all $\mathbf{R}^1, \mathbf{R}^2 \in \mathcal{D}$, if for all $i \in N$, $[(x, A)R_i^1(y, B) \Leftrightarrow (z, C)R_i^2(w, D)$ and $(y, B)R_i^1(x, A) \Leftrightarrow (w, D)R_i^2(z, C)]$, then $[(x, A)R^1(y, B) \Leftrightarrow (z, C)R^2(w, D)$ and $(y, B)R^1(x, A) \Leftrightarrow (w, D)R^2(z, C)]$, where $R^1 = f(\mathbf{R}^1)$ and $R^2 = f(\mathbf{R}^2)$.

(A) requires that all individuals are treated symmetrically or equally, that is, an ESWF does not depend on which individual expresses which extended preference. For restricted domains, (A) does not always imply (ND).[6] (N) requires that an ESWF should treat all extended alternatives symmetrically. Note that (N) implies (IIA).

We show that if we additionally impose (A) or (N) on an ESWF, then there exists no ESWF defined on $\mathcal{D}_{ES}$ which satisfies (WP), (IIA) and (ND).

Theorem 7 There exists no extended social welfare function defined on $\mathcal{D}_{ES}$ which satisfies (WP), (IIA), (ND) and (A).

Theorem 8 There exists no extended social welfare function defined on $\mathcal{D}_{ES}$ which satisfies (WP), (N) and (ND).

7 Concluding Remarks

We examined whether it is possible to resolve Arrow's impossibility theorem within consequentialism. We first have proved that Arrow's impossibility theorem holds on a class of domains. If a domain is such that all individuals are either extreme consequentialists or strong consequentialists, then it is in the proposed class of domains. On the other hand, If there simultaneously exist one extreme consequentialist and one strong consequentialist in society, then it is possible to construct an ESWF which satisfies weak Pareto, independence of irrevant alternatives and nondictatorship. In this sense the diversity of choice attitude towards outcomes and opportunitites plays a crucial role in resoluving Arrow's impossibility theorem. This statement is basically the same message as Suzumura and Xu's (2004).

In addition, we imposed Pareto indifference, Anonymity or Neutrality on an ESWF, and proved that there exists no ESWF which satisfies Arrow's conditions and the above additional conditions, even if there simultaneously exist an exterme consequentialist and a strong consequentialist in society. Our results contains an amendment to a Suzumura and Xu's (2004) theorem and explains a boundary between the possibility and the impossibility of social choice within consequentialism.

8 Appendix

Proof of Theorem 1

Conditions (2) and (3) imply that for all $\{(x, A), (y, B)\} \subset \Omega$ and $\{(z, C), (w, D)\} \subset \Omega$, if the two pairs are CRO nontrivial with respect to $\mathcal{D}$, then there exist $(x^1, A^1), (x^2, A^2), \ldots,$

[6] See Sakai and Shimoji (2006) for detail.

$(x^T, A^T) \in \Omega$ such that:

$$(x^1, A^1) = (x, A), (x^2, A^2) = (y, B), (x^{T-1}, A^{T-1}) = (z, C), (x^T, A^T) = (w, D);$$

and:

for all t from 1 to $T-2$:

$$\Gamma^t = \{(x^t, A^t), (x^{t+1}, A^{t+1}), (x^{t+2}, A^{t+2})\}$$

is a triple which is CRO free with respect to $\mathcal{D}$.

Let $\{(x^1, A^1), (x^2, A^2)\}$ be a CRO nontrivial pair with respect to $\mathcal{D}$. Then it belongs to a triple Γ^1 which is CRO free with respect to $\mathcal{D}$, and on which by condition (1), $\mathcal{D}$ is locally an A-domain. By (IIA) and Lemma 1, there exists a local dictator, say d_1, on this triple. Let $\{(x^{T-1}, A^{T-1}), (x^T, A^T)\}$ be any other pair which is CRO nontrivial with respect to $\mathcal{D}$. By the above argument, we get the triples $\Gamma^1, \Gamma^2, \ldots, \Gamma^{T-2}$, with $\{(x^{T-1}, A^{T-1}), (x^T, A^T)\} \in \Gamma^{T-2}$. By the same reasoning as above, there exists a local dictator d_t on each of the Γ^t, and hence a local dictator d_{T-2} on the second pair. But, for all t: $\sharp(\Gamma^t \cap \Gamma^{t+1}) = 2$, and hence obviously $d_t = d_{t+1}$, and so $d_1 = d_{T-2}$, and we have the same local dictator, say d, on all CRO nontrivial pairs with respect to $\mathcal{D}$.

Note that it follows from Domain Assumption that a pair is CRO trivial with respect to $\mathcal{D}$ if and only if it is trivial with respect to $\mathcal{D}$. Since $\mathcal{D}$ is a common domain, all individuals have the same extended preference orderings over any trivial pair with respect to $\mathcal{D}$. (WP) implies that d is a dictator over all trivial pairs with respect to $\mathcal{D}$. Thus, d is a dictator over Ω. ■

Proof of Proposition 1

We first show that $\mathcal{D}_E$ satisfies Domain Assumption. Suppose that for all pairs $\{(x, A), (y, B)\} \subset \Omega$, it is CRO nontrivial with respect to $\mathcal{D}_E$. Note that for all pairs $\{(x, A), (y, B)\} \subset \Omega$, it is CRO nontrivial with respect to $\mathcal{D}_E$ if and only if $x \neq y$. If $x \neq y$, then an extreme consequentialist does not impose any restrictions on extended preference orderings between $(x, \{x\})$ and $(y, \{y\})$. Therefore, the pair $\{(x, A), (y, B)\}$ is nontrivial with respect to $\mathcal{D}_E$ by the definition of $\mathcal{D}_E$. The converse can be shown by an analogical argument.

For $\mathcal{D}_S$ and $\mathcal{D}_{ES}$, we can show that they satisfy Domain Assumption by a similar reasoning to $\mathcal{D}_E$. ■

Proof of Proposition 2

It is clear that the domains $\mathcal{D}_E$ and $\mathcal{D}_S$ are common. We have to show that the domains $\mathcal{D}_E$ and $\mathcal{D}_S$ satisfy the three conditions defining extendedly saturating domains. We first show that $\mathcal{D}_E$ is extendedly saturating.

Condition (1). Note that $\mathcal{D}_E = \mathcal{D}_e \times \cdots \times \mathcal{D}_e$. For all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_e$ if and only if x, y and z are all distinct. Since $\mathcal{D}_e$ is not imposed any restrictions on extended preference orderings among $(x, \{x\})$, $(y, \{y\})$ and $(z, \{z\})$, it is clear that $L(\Omega)|_\Gamma \subset \mathcal{D}_e|_\Gamma = P(\Omega)|_\Gamma$ by the definition of $\mathcal{D}_e$, which implies that $\mathcal{D}_e$ is locally an I-domain. Furthermore, it follows from $\mathcal{D}_e|_\Gamma = P(\Omega)|_\Gamma$ that $\mathcal{D}_E$ is locally an A-domain.

Condition (2). For all pairs $\{(x, A), (y, B)\} \subset \Omega$, it is CRO nontrivial with respect to $\mathcal{D}_E$ if and only if $x \neq y$. We have four distinguished cases: (a) $x = z$ and $y = w$; (b) $x = z$ and $y \neq w$; (c) $x \neq z$ and $y = w$; and (d) $x \neq z$ and $y \neq w$.

Case (a): In this case, there exists some $(v, E) \in \Omega$ such that $x \neq v \neq y$. Consider the following sequence of pairs: $\{(x, A), (y, B)\}$, $\{(y, B), (v, E)\}$, $\{(v, E), (z, C)\}$ and $\{(z, C), (w, D)\}$. Since for all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_E$ if and only if x, y and z are all distinct, there exists a sequence such that adjoining pairs are CRO free with respect to $\mathcal{D}_E$.

Case (b): In this case, consider the following sequence of pairs: $\{(x, A), (y, B)\}$, $\{(y, B), (w, D)\}$ and $\{(w, D), (z, C)\}$. Since for all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_E$ if and only if x, y and z are all distinct, there exists a sequence such that adjoining pairs are CRO free with respect to $\mathcal{D}_E$.

Case (c): In this case, consider the following sequence of pairs: $\{(y, B), (x, A)\}$, $\{(x, A), (z, C)\}$ and $\{(z, C), (w, D)\}$. Since for all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_E$ if and only if x, y and z are all distinct, there exists a sequence such that adjoining pairs are CRO free with respect to $\mathcal{D}_E$.

Case (d): In this case, consider the following sequence of pairs: $\{(x, A), (y, B)\}$, $\{(y, B), (z, C)\}$ and $\{(z, C), (w, D)\}$. Since for all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_E$ if and only if x, y and z are all distinct, there exists a sequence such that adjoining pairs are CRO free with respect to $\mathcal{D}_E$.

Combining from (a) to (d), we prove that $\mathcal{D}_E$ satisfies condition (2).

Condition (3). By $\sharp X \geq 3$, $\mathcal{D}_E$ clearly satisfies condition (3).

Next, we show that $\mathcal{D}_S$ is extendedly saturating. Conditions (2) and (3) can be shown by a similar argument to $\mathcal{D}_E$. We have to show that $\mathcal{D}_S$ satisfies condition (1).

Condition (1). Note that $\mathcal{D}_S = \mathcal{D}_s \times \cdots \times \mathcal{D}_s$. For all triples $\Gamma = \{(x, A), (y, B), (z, C)\} \subset \Omega$, it is CRO free with respect to $\mathcal{D}_s$ if and only if x, y and z are all distinct. Since $\mathcal{D}_s$ is not imposed any restrictions on extended preference orderings among $(x, \{x\})$, $(y, \{y\})$ and $(z, \{z\})$, it is clear that $L(\Omega)|_\Gamma \subset \mathcal{D}_s|_\Gamma \subset P(\Omega)|_\Gamma$ by the definition of $\mathcal{D}_s$, which implies that $\mathcal{D}_S$ is locally an I-domain.

Then we have to show that $\mathcal{D}_S$ is locally an A-domain. If $R_s^*|_\Gamma$ belongs to $\mathcal{D}_s|_\Gamma$, then we have to have $\sharp A = \sharp B = \sharp C$, where $R_s^*|_\Gamma$ is such that $(x, A)I_s^*(y, B)I_s^*(z, C)$. By the definition of $\mathcal{D}_s$, Γ must be free with respect to $\mathcal{D}_s$, which implies that $\mathcal{D}_S$ is locally an A-domain. If $R_s^*|_\Gamma$ does not belong to $\mathcal{D}_s|_\Gamma$, then $\mathcal{D}_S$ is locally an A-domain. ■

Proof of Theorem 4

Given an outcome x^*, consider the following five subsets of Ω: $\Omega_1 = \{(x^*, X)\}$, $\Omega_2 = \{(x, A) \in \Omega \mid x = x^* \text{ and } \sharp A = \sharp X - 1\}$, $\Omega_3 = \{(x, A) \in \Omega \mid x \neq x^* \text{ and } \sharp A = \sharp X\}$, $\Omega_4 = \{(x, A) \in \Omega \mid x \neq x^* \text{ and } \sharp A = \sharp X - 1\}$ and $\Omega_5 = \Omega \setminus \bigcup_{i=1}^{4} \Omega_i$. Note that for all $i, j \in \{1, \ldots, 5\}$, $i \neq j$, Ω_i and Ω_j are disjoint.

Then consider the following ESWF: For all $(x, A), (y, B) \in \Omega$ and all $\mathbf{R} \in \mathcal{D}_{ES}$,
if $(x, A) \in \Omega_1$ and $(y, B) \in \Omega_2$, then

$$(x, A)I(y, B); \tag{1}$$

if $(x, A) \in \Omega_1$, $(y, B) \in \Omega_3$ and $(x, \{x\})I_s(y, \{y\})$, then

$$(y, B)P(x, A); \tag{2}$$

if $(x, A) \in \Omega_2$, $(y, B) \in \Omega_4$ and $(x, \{x\})I_s(y, \{y\})$, then

$$(x, A)P(y, B); \tag{3}$$

otherwise,

$$(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B), \tag{4}$$

where $R = f(\mathbf{R})$. By construction, this ESWF satisfies (WP), (IIA) and (ND).

The binary relation R generated by this ESWF is clearly reflexive and complete. We now show that R is transitive. Suppose, to the contrary, that R is not transitive. That is, there exist (x, A), (y, B) and $(z, C) \in \Omega$, and a profile $\mathbf{R} \in \mathcal{D}_{ES}$ such that $(x, A)R(y, B)$ and $(y, B)R(z, C)$ but $(z, C)P(x, A)$, where $R = f(\mathbf{R})$. Since R is reflexive and complete, we have only to consider the case where (x, A), (y, B) and (z, C) are all distinct. By $(z, C)P(x, A)$, we have four possible cases: (a) $(x, A) \in \Omega_1$, $(z, C) \in \Omega_3$ and $(x\{x\})I_s(z, \{z\})$; (b) $(x, A) \in \Omega_4$, $(z, C) \in \Omega_2$ and $(x, \{x\})I_s(z, \{z\})$; (c) $(x, A)R(z, C) \Leftrightarrow (x, A)R_s(z, C)$ and $(z, \{z\})P_s(x, \{x\})$; and (d) $(x, A)R(z, C) \Leftrightarrow (x, A)R_s(z, C)$, $(x, \{x\})I_s(z, \{z\})$ and $\sharp A < \sharp C$.

Case (a): We distinguish four subcases (a-1) $(y, B) \in \Omega_2$; (a-2) $(y, B) \in \Omega_3$; (a-3) $(y, B) \in \Omega_4$; and (a-4) $(y, B) \in \Omega_5$.

Subcase (a-1): We have $(y, B)R(z, C) \Leftrightarrow (y, B)R_s(z, C)$ by (4). It follows from $(y, B)R(z, C)$ that $(y, B)R_s(z, C)$. Since $(x, A) \in \Omega_1$, $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_3$, we have $x = y = x^*$

and $\sharp B < \sharp C$. Given the assumption in case (a), $(x, \{x\}) I_s (z, \{z\})$, we can get $(y, \{y\}) I_s (z, \{z\})$. $(z, C) P_s (y, B)$ follows from $(y, \{y\}) I_s (z, \{z\})$ and $\sharp B < \sharp C$, which is a contradiction to $(y, B) R_s (z, C)$.

Subcase (a-2): If $(x, \{x\}) I_s (y, \{y\})$, then we must have $(y, B) P (x, A)$ by (2), which is a contradiction to $(x, A) R (y, B)$. If $\neg[(x, \{x\}) I_s (y, \{y\})]$, then we have $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ by (4). $(x, A) R (y, B)$ implies $(x, \{x\}) P_s (y, \{y\})$. Given the assumption in case (a), $(x, \{x\}) I_s (z, \{z\})$, we have $(z, \{z\}) P_s (y, \{y\})$ by the transitivity of R_s. It follows from $(z, \{z\}) P_s (y, \{y\})$ that $(z, C) P_s (y, B)$. Since $(y, B) \in \Omega_3$ and $(z, C) \in \Omega_3$, we have $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$ by (4). $(y, B) R (z, C)$ implies $(y, B) R_s (z, C)$, which leads to a contradiction to $(z, C) P_s (y, B)$.

Subcase (a-3): We have $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ by (4). By $(x, A) R (y, B)$, we obtain $(x, A) R_s (y, B)$. $(x, A) R_s (y, B)$ implies $(x, \{x\}) R_s (y, \{y\})$. Given the assumption in case (a), $(x, \{x\}) I_s (z, \{z\})$, we have $(z, \{z\}) R_s (y, \{y\})$ by the transitivity of R_s. Since $(y, B) \in \Omega_4$ and $(z, C) \in \Omega_3$, we have $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$ by (4), and $\sharp B < \sharp C$. By $(z, \{z\}) R_s (y, \{y\})$ and $\sharp B < \sharp C$, we have $(z, C) P_s (y, B)$. However, $(y, B) R (z, C)$ implies $(y, B) R_s (z, C)$, which leads to a contradiction to $(z, C) P_s (y, B)$.

Subcase (a-4): We can lead to a similar contradiction to subcase (a-3).

Case (b): We distinguish five subcases (b-1) $(y, B) \in \Omega_1$; (b-2) $(y, B) \in \Omega_2$; (b-3) $(y, B) \in \Omega_3$; (b-4) $(y, B) \in \Omega_4$; and (b-5) $(y, B) \in \Omega_5$.

Subcase (b-1): We have $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ by (4). By $(x, A) R (y, B)$, we have $(x, A) R_s (y, B)$. Since $(x, A) \in \Omega_4$, $(y, B) \in \Omega_1$ and $(z, C) \in \Omega_2$, we have $y = z = x^*$ and $\sharp A < \sharp B$. By the assumption in case (b), $(x, \{x\}) I_s (z, \{z\})$, we have $(x, \{x\}) I_s (y, \{y\})$. However, $(x, \{x\}) I_s (y, \{y\})$ and $\sharp A < \sharp B$ imply $(y, B) P_s (x, A)$, which contradicts to $(x, A) R_s (y, B)$.

Subcase (b-2): Since $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_2$, we have $y = z = x^*$. By the assumption in case (b), $(x, \{x\}) I_s (z, \{z\})$, we have $(x, \{x\}) I_s (y, \{y\})$. Since $(x, A) \in \Omega_4$, we have $(y, B) P (x, A)$ by (3), which contradicts to $(x, A) R (y, B)$.

Subcase (b-3): We have $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ by (4). Since $(x, A) \in \Omega_4$ and $(y, B) \in \Omega_3$, we have $\sharp A < \sharp B$. Given $\sharp A < \sharp B$, $(x, A) R (y, B)$ implies $(x, \{x\}) P_s (y, \{y\})$. Given the assumption in case (b), $(x, \{x\}) I_s (z, \{z\})$, we have $(z, \{z\}) P_s (y, \{y\})$ by the transitivity of R_s. $(z, \{z\}) P_s (y, \{y\})$ implies $(z, C) P_s (y, B)$. Since $(y, B) \in \Omega_3$ and $(z, C) \in \Omega_2$, we have $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$. $(y, B) R (z, C)$ implies $(y, B) R_s (z, C)$, which contradicts to $(z, C) P_s (y, B)$.

Subcase (b-4): We have $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ by (4). It follows from $(x, A) R (y, B)$ that $(x, A) R_s (y, B)$. If $(y, \{y\}) I_s (z, \{z\})$, then we have $(z, C) P (y, B)$ by (3), which is a contradiction to $(y, B) R (z, C)$. Therefore, we have $\neg[(y, \{y\}) I_s (z, \{z\})]$. Then $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$ by (4). By $(y, B) R (z, C)$, we have $(y, B) R_s (z, C)$. Since $\neg[(y, \{y\}) I_s (z, \{z\})]$, we must have $(y, \{y\}) P_s (z, \{z\})$. Given the assumption in case (b), $(x, \{x\}) I_s (z, \{z\})$, we have $(y, \{y\}) P_s (x, \{x\})$ by the transitivity of R_s. However, $(y, \{y\}) P_s (x, \{x\})$ implies $(y, B) P_s (x, A)$, which leads to a contradiction to $(x, A) R_s (y, B)$.

Subcase (b-5): We can lead a similar contradiction to subcase (a-3).

Case (c): Suppose that $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ and $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$ by (4). By $(x, A) R (y, B)$, we have $(x, A) R_s (y, B)$. We also have $(y, B) R_s (z, C)$ by $(y, B) R (z, C)$. It follows from the transitivity of R_s that $(x, A) R_s (z, C)$. However, we have $(z, C) P_s (x, A)$ in this case, which is a contradiction to $(x, A) R_s (z, C)$. Hence, $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ and $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$ do not hold simultaneously.

Suppose that $(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)$ and $\neg[(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)]$. We have $(x, A) R_s (y, B)$ by $(x, A) R (y, B)$. It follows from $(x, A) R_s (y, B)$ that $(x, \{x\}) R_s (y, \{y\})$. If $(y, B) R (z, C)$ is determined by (2) or (3), then we have $(y, \{y\}) I_s (z, \{z\})$. It follows from the transitivity of R_s that $(x, \{x\}) R_s (z, \{z\})$, which leads to a contradiction to $(z, \{z\}) P_s (x, \{x\})$. Therefore, we must have $(y, B) I (z, C)$ by (1), which implies $y = z = x^*$. However, we have $(x, \{x\}) R_s (z, \{z\})$ by $(x, \{x\}) R_s (y, \{y\})$, which leads to a contradiction to $(z, \{z\}) P_s (x, \{x\})$.

If we suppose that $\neg[(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)]$ and $(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)$, then we can lead to a similar contradiction above. Thus, we must have $\neg[(x, A) R (y, B) \Leftrightarrow (x, A) R_s (y, B)]$ and $\neg[(y, B) R (z, C) \Leftrightarrow (y, B) R_s (z, C)]$.

Suppose $(x, A) I (y, B)$ and $(y, B) I (z, C)$ by (1). It follows from (1) that $x = y = z = x^*$. However, $x = z$ implies $(x, \{x\}) I_s (z, \{z\})$, which is a contradiction to $(z, \{z\}) P_s (x, \{x\})$. Therefore, $(x, A) I (y, B)$ and $(y, B) I (z, C)$ do not simultaneously hold by (1).

Suppose that $(x, A) I (y, B)$ by (1) and $\neg[(y, B) I (z, C)]$. By (1), we have either $(y, B) \in \Omega_1$ or $(y, B) \in \Omega_2$. If $(y, B) R (z, C)$ is determined by (2), then we get $(y, B) \in \Omega_3$ and $(z, C) \in \Omega_1$ by $(y, B) R (z, C)$. However, $(y, B) \in \Omega_3$ contradicts to either $(y, B) \in \Omega_1$ or $(y, B) \in \Omega_2$. If $(y, B) R (z, C)$ is determined by (3), then we obtain $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_4$ by $(y, B) R (z, C)$, and $(y, \{y\}) I_s (z, \{z\})$. Since $(x, A) I (y, B)$ by (1), we have $x = y = x^*$. Therefore, we have $(x, \{x\}) I_s (z, \{z\})$ by $(y, \{y\}) I_s (z, \{z\})$, which leads to a contradiction to $(z, \{z\}) P_s (x, \{x\})$.

If we suppose $\neg[(x, A) I (y, B)]$ and $(y, B) I (z, C)$, then we can lead to a similar contradiction above. Thus, we must have $\neg[(x, A) I (y, B)]$ and $\neg[(y, B) I (z, C)]$.

If $(x, A) R (y, B)$ is determined by (2), then it follows from $(x, A) R (y, B)$ that $(x, A) \in \Omega_3$ and $(y, B) \in \Omega_1$. Further, if $(y, B) R (z, C)$ is determined by (2), then it follows from $(y, B) R (z, C)$ that $(y, B) \in \Omega_3$ and $(z, C) \in \Omega_1$. However, $(y, B) \in \Omega_3$ contradicts to $(y, B) \in \Omega_1$. Therefore, $(y, B) R (z, C)$ must be determined by (3). It follows from $(y, B) R (z, C)$ that $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_4$. However, $(y, B) \in \Omega_2$ contradicts to $(y, B) \in \Omega_1$.

If we suppose that $(x, A)R(y, B)$ is determined by (3), then we can lead to a similar contradiction above.

Case (d): By the same reasoning as case (c), $(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)$ and $(y, B)R(z, C) \Leftrightarrow (y, B)R_s(z, C)$ do not hold simultaneously.

Suppose that $(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)$ and $\neg[(y, B)R(z, C) \Leftrightarrow (y, B)R_s(z, C)]$. We have $(x, A)R_s(y, B)$ by $(x, A)R(y, B)$. If $(y, B)R(z, C)$ is determined by (2), then we have $(y, B) \in \Omega_3$, $(z, C) \in \Omega_1$ and $(y, \{y\})I_s(z, \{z\})$. By $(y, B) \in \Omega_3$ and $(z, C) \in \Omega_1$ we have $\sharp B = \sharp C = \sharp X$. Given the assumption in case (d), $(x, \{x\})I_s(z, \{z\})$, we have $(x, \{x\})I_s(y, \{y\})$ by the transitivity of R_s. Again, given the assumption in case (d), $\sharp A < \sharp C$, we have $\sharp A < \sharp B$. $(y, B)P_s(x, A)$ follows from $(x, \{x\})I_s(y, \{y\})$ and $\sharp A < \sharp B$, which is a contradiction to $(x, A)R_s(y, B)$. If $(y, B)R(z, C)$ is determined by (3), then we can show a similar contradiction to (2). Therefore, we must have $(y, B)I(z, C)$ by (1). By (1), we obtain $y = z = x^*$. By the assumption in case (d), $(x, \{x\})I_s(z, \{z\})$, we can get $(x, \{x\})I_s(y, \{y\})$. Suppose $\sharp A < \sharp B$. Then we have $(y, B)P_s(x, A)$ by $(x, \{x\})I_s(y, \{y\})$ and $\sharp A < \sharp B$, which is a contradiction to $(x, A)R_s(y, B)$. Therefore, given the assumption in case (d), $\sharp A < \sharp C$, we have $\sharp B \leq \sharp A < \sharp C$, which implies $\sharp A = \sharp X - 1$, $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_1$. If $x = x^*$, then $(x, A) \in \Omega_2$. Since $(x, A) \in \Omega_2$ and $(z, C) \in \Omega_1$, we have $(x, A)I(z, C)$ by (1), which is a contradiction to $(z, C)P(x, A)$. If $x \neq x^*$, then $(x, A) \in \Omega_4$. Since $(x, A) \in \Omega_4$, $(y, B) \in \Omega_2$ and $(x, \{x\})I_s(y, \{y\})$, we have $(y, B)P(x, A)$ by (3), which leads to a contradiction to $(x, A)R(y, B)$.

If we suppose that $\neg[(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)]$ and $(y, B)R(z, C) \Leftrightarrow (y, B)R_s(z, C)$, we can lead to a similar contradiction above. Thus, we must have $\neg[(x, A)R(y, B) \Leftrightarrow (x, A)R_s(y, B)]$ and $\neg[(y, B)R(z, C) \Leftrightarrow (y, B)R_s(z, C)]$.

Suppose that $(x, A)I(y, B)$ and $(y, B)I(z, C)$ by (1). Given the assumption in case (d), $\sharp A < \sharp C$, we obtain $(x, A) \in \Omega_2$ and $(y, B) \in \Omega_1$. Since $(y, B)I(z, C)$ by (1), we have $(z, C) \in \Omega_2$ because of $(y, B) \in \Omega_1$. We have $\sharp A = \sharp C$ by $(x, A) \in \Omega_2$ and $(z, C) \in \Omega_2$, which is a contradiction to $\sharp A < \sharp C$. Thus, $(x, A)I(y, B)$ and $(y, B)I(z, C)$ do not hold simultaneously by (1).

Suppose that $(x, A)I(y, B)$ by (1) and $\neg[(y, B)I(z, C)]$. If $(y, B)R(z, C)$ is determined by (2), then we can lead to a contradiction by a similar proof to case (c). If $(y, B)R(z, C)$ is determined by (3), then we have $(y, B) \in \Omega_2$ and $(z, C) \in \Omega_4$ by $(y, B)R(z, C)$, Since $(x, A)I(y, B)$ by (1), we have $(x, A) \in \Omega_1$ because of $(y, B) \in \Omega_2$. Since $(x, A) \in \Omega_1$ and $(z, C) \in \Omega_4$, we have $\sharp A = \sharp X$ and $\sharp C = \sharp X - 1$, which contradicts to $\sharp A < \sharp C$.

If we suppose that $\neg[(x, A)I(y, B)]$ and $(y, B)I(z, C)$ by (1), then we can lead to a similar contradiction above. Therefore, we must have $\neg[(x, A)I(y, B)]$ and $\neg[(y, B)I(z, C)]$.

If $(x, A)R(y, B)$ is determined by (2) or (3), we can show a similar contradiction as case (c).

Combining from case (a) to case (d), we obtain the transitivity of R. ■

Proof of Theorem 5

Since $\mathcal{D}$ is extendedly saturating, we can show that there exists a dictator on the set of CRO nontrivial pairs with respect to $\mathcal{D}$ by Theorem 1.

Without loss of generality, suppose that individual d is the dictator on the CRO nontrivial pairs with respect to $\mathcal{D}$. Let $(R_1, \ldots, R_n) \in \mathcal{D}$ and $\{(x, A), (y, B)\} \subset \Omega$ be such that for at least one $i \in N$, the pair is CRO trivial with respect to $\mathcal{D}_i$, and $(x, A)P_d(y, B)$. (If no such profile and pair exist, we have the result.) We have to show that $(x, A)P(y, B)$.

We consider a CRO trivial pair $\{(x, A), (y, B)\} \subset \Omega$ with respect to $\mathcal{D}$. Let $\mathbf{R}^1 \in \mathcal{D}$ be any profile with $(x, A)P_d(y, B)$:

$$\begin{cases} (x, A)P_d^1(y, B) \\ (x, A)P_i^1(y, B), \quad \forall i \in S_1 \\ (y, B)P_i^1(x, A), \quad \forall i \in S_2 \\ (x, A)I_i^1(y, B), \quad \forall i \in S_3 \end{cases}$$

where some among S_1, S_2, S_3 may be empty. Since $\mathcal{D}$ is extendedly strongly saturating, there exists a profile $\mathbf{R}^2 \in \mathcal{D}$ with

$$\begin{cases} (x, A)I_d^2(z, C)P_d^2(y, B) \\ (x, A)I_i^2(z, C)P_i^2(y, B), \quad \forall i \in S_1 \\ (y, B)P_i^2(z, C)I_i^2(x, A), \quad \forall i \in S_2 \\ (x, A)I_i^2(z, C)I_i^2(y, B), \quad \forall i \in S_3 \end{cases}$$

Since $\{(y, B), (z, C)\}$ is CRO nontrivial with respect to $\mathcal{D}$, we have $(z, C)P^2(y, B)$ by the dictatorship of d over $\{(y, B), (z, C)\}$, where $R^2 = f(\mathbf{R}^2)$. $(x, A)I^2(z, C)$ follows by (PI), where $R^2 = f(\mathbf{R}^2)$. Therefore, the transitivity of R implies $(x, A)P^2(y, B)$. By (IIA), $(x, A)P^1(y, B)$, where $R^1 = f(\mathbf{R}^1)$, and so d is a dictator on $\{(x, A), (y, B)\}$. Thus, d is a dictator on all of Ω. ■

Proof of Proposition 3

We have to show that the domain $\mathcal{D}_{ES}$ satisfies the two conditions defining extendedly strongly saturating domains.

We can show that $\mathcal{D}_{ES}$ is extendedly saturating by a similar argument to the proof of Proposition 2. Therefore, $\mathcal{D}_{ES}$ satisfies condition (a) in extendedly strongly saturating domains. From now on, we show that $\mathcal{D}_{ES}$ satisfies condition (b). Note that for all pairs $\{(x, A), (y, B)\} \subset \Omega$, it is CRO trivial with respect to $\mathcal{D}_{ES}$ if and only if $x = y$. We consider an extended alternative (z, C) such that $x = y \neq z$ and $\sharp A = \sharp C$. For all $i \in N \backslash \{e, s\}$, the requisite extended preference clearly exists. The pair $\{(x,A),(y,B)\}$ is CRO trivial with respect to $\mathcal{D}_e$ and we have $(x,A)I_e(y,B)$ for all $R_e \in \mathcal{D}_e$. Since for all pairs $\{(x,A),(y,B)\} \subset \Omega$, it is CRO nontrivial with respect to $\mathcal{D}_e$ if and only if $x \neq y$, $\{(x,A),(z,C)\}$ and $\{(y,B),(z,C)\}$ are CRO nontrivial with respect to $\mathcal{D}_e$. It is easy to confirm that there exists an extended preference $R_e^{'}$ such that $(x,A)I_e^{'}(z,C)I_e^{'}(y,B)$.

The pair $\{(x,A),(y,B)\}$ is CRO trivial with respect to $\mathcal{D}_s$. We distinguish three cases: (a) $(x,A)P_s(y,B)$ for all $R_s \in D_s$ if and only if $\sharp A > \sharp B$; (b) $(y,B)P_s(x,A)$ for all $R_s \in \mathcal{D}_s$ if and only if $\sharp B > \sharp A$; and (c) $(x,A)I_s(y,B)$ for all $R_s \in \mathcal{D}_s$ if and only if $\sharp A = \sharp B$.

Case (a) Since for all pairs $\{(x,A),(y,B)\} \subset \Omega$, it is CRO nontrivial with respect to $\mathcal{D}_s$ if and only if $x \neq y$, $\{(x,A),(z,C)\}$ and $\{(y,B),(z,C)\}$ are CRO nontrivial with respect to $\mathcal{D}_s$. It is easy to confirm that there exists an extended preference $R_s^{'}$ such that $(x,A)I_s^{'}(z,C)P_s^{'}(y,B)$.

In cases (b) and (c), we can prove by a similar argument to case (a). ■

Proof of Theorem 7

Consider a triple $\{(x,A),(y,B),(z,C)\}$ such that x,y and z are all distinct and $\sharp A = \sharp B = \sharp C$. Then $\{(x,A),(y,B),(z,C)\}$ is a free triple with respect to $\mathcal{D}_{ES}$. Therefore, there exists a local dictator on $\{(x,A),(y,B),(z,C)\}$. Without loss of generality individual d is a local dictator on $\{(x,A),(y,B),(z,C)\}$.

Consider any profile $\mathbf{R}^1$ such that $(x,A)P_d^1(y,B)$ and $(y,B)P_i^1(x,A)$ for an indvidual $i \in N$. Since d is a local dictator on $\{(x,A),(y,B),(z,C)\}$, we have $(x,A)P^1(y,B)$, where $R^1 = f(\mathbf{R}^1)$. Since $\{(x,A),(y,B),(z,C)\}$ is free with respect to $\mathcal{D}_{ES}$, there exists the following profile $\mathbf{R}^2$: $(y,B)P_d^2(x,A)$, $(x,A)P_i^2(y,B)$ and $(x,A)R_j^1(y,B) \Leftrightarrow (x,A)R_j^2(y,B)$ for all $j \in N\backslash\{d,i\}$. Since there exists a π such that $\mathbf{R}^2 = \pi(\mathbf{R}^1)$, we have $(x,A)P^2(y,B)$ by (A), where $R^2 = f(\mathbf{R}^2)$. However, since d is a local dictatior on $\{(x,A),(y,B),(z,C)\}$, we must have $(y,B)P^2(x,A)$, which is a contradiction. ■

Proof of Theorem 8

Since $\mathcal{D}_{ES}$ is extendedly saturating by Proposition 3, there exists a dictator on the set of pairs which are CRO nontrivial with respect to $\mathcal{D}_{ES}$ by Theorem 1.

Without loss of generality, suppose that individual d is the dictator on the CRO nontrivial pairs with respect to $\mathcal{D}_{ES}$. Let $(R_1,\ldots,R_n) \in \mathcal{D}_{ES}$ and $\{(x,A),(y,B)\} \subset \Omega$ be such that for at least one $i \in N$, the pair is CRO trivial with respect to $\mathcal{D}_i$, and $(x,A)P_d(y,B)$. (If no such pair and profile exist, we have the result.) We have to show that $(x,A)P(y,B)$.

Consider a pair $\{(x,A),(y,B)\}$, which is CRO trivial with respect to $\mathcal{D}_{ES}$. Let $\mathbf{R}^1 \in \mathcal{D}_{ES}$ be any profile with $(x,A)P_d(y,B)$:

$$\begin{cases} (x,A)P_d^1(y,B) \\ (x,A)P_i^1(y,B), \quad \forall i \in S_1 \\ (y,B)P_i^1(x,A), \quad \forall i \in S_2 \\ (x,A)I_i^1(y,B), \quad \forall i \in S_3 \end{cases}$$

where some among S_1, S_2, S_3 may be empty. Let us consider a CRO nontrivial pair $\{(z,C),(w,D)\}$ with respect to $\mathcal{D}_{ES}$ such that $z \neq w$ and $\sharp C = \sharp D$. Then this pair in fact is a free pair with respect to $\mathcal{D}_{ES}$. Therefore there exists a profile $\mathbf{R}^2 \in \mathcal{D}_{ES}$ with:

$$\begin{cases} (z,C)P_d^2(w,D) \\ (z,C)P_i^2(w,D), \quad \forall i \in S_1 \\ (w,D)P_i^2(z,C), \quad \forall i \in S_2 \\ (z,C)I_i^2(w,D), \quad \forall i \in S_3 \end{cases}$$

Since d is a dictator on CRO nontrivial pairs, we have $(z,C)P^2(w,D)$, where $R^2 = f(\mathbf{R}^2)$. Since for all $i \in N$, $(x,A)R_i^1(y,B) \Leftrightarrow (z,C)R_i^2(w,D)$, it follows from (N) that $(x,A)P^1(y,B)$, where $R^1 = f(\mathbf{R}^1)$. and so d is a dictator on $\{(x,A),(y,B)\}$. Thus, d is a dictator on all of Ω. ■

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