We consider *direct revelation mechanisms*, which consist of an allocation rule that selects an allocation depending on the agents' reports about their private information, and a payment scheme that assigns a payment to every agent. Allocation rules that give rise to a mechanism in which truth-telling is a dominant strategy for every agent are called *truthfully implementable*. Our concern is with the payment scheme that extends a truthfully implementable allocation rule to a truthful mechanism. The property of an allocation rule to have a unique payment scheme completing the allocation rule to a truthful mechanism is called *revenue equivalence*.

We give a characterization for an allocation rule to satisfy revenue equivalence. In order to obtain this characterization, we prove a property on complete directed graphs and apply it to the so called *allocation graph*, which is defined by the allocation rule and the valuation function of an agent. The characterization holds for any outcome space, and can thus also be used to characterize revenue equivalence for Bayesian-Nash incentive compatible allocation rules. Furthermore, we give elementary and simple proofs for revenue equivalence under very weak assumptions in two special cases: finite outcome spaces and countable infinite outcome spaces. For these cases, our results extend the well-know results by Green and Laffont [1] and Holmström [2] on the uniqueness of VCG mechanisms, and Milgrom and Segal [4] and Krishna and Maenner [3] on revenue equivalence for arbitrary allocation rules. In particular, the conditions for which revenue equivalence holds are much weaker and easy to verify.

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