

Some Results on the Cores in an Economy with Indivisible Commodities

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Abstract

I introduce some results obtained in my papers [1] and [2]. In both papers, I consider a finite exchange economy where every commodity is available only in integer amounts. Therefore, the commodity space is given by the products of the set of integers. In this economy, because of the satiation of agents' preference relations, the size of the cores depends heavily on the improvement defining the cores. The strong core is defined by the weak improvement; some members in a coalition can be better off and other members cannot be worse off. On the other hand, the weak core is defined by the strong improvement; all members in a coalition can be better off. By definition, the strong core is a subset of the weak core.

In [1], I show that in an economy with sufficiently large agents, but finitely many agents, every strong core, if it exists, can be supported by a Walras equilibrium. Because of the indivisibility, the strong core and Walras equilibrium may not exist. I have not obtained a sufficient condition for the strong core and the Walras equilibrium to exist.

In [2], I show that if the aggregate upper contour set is discretely convex, then the weak core is nonempty. The discrete convexity used in [2] is not closed under summation. Thus, even if every agent's preference relation is discretely convex, the aggregate upper contour set may not be discretely convex. But, if every agent's preference relation is M^{\sharp} -convex (a more restrictive convexity than the discrete convexity), then the aggregate upper contour set is discretely convex. Therefore, if every agent's preference relation is M^{\sharp} -convex, then the weak core is nonempty. Agents' preference relations of Shapley-Scarf's house swapping market game are M^{\sharp} -convex.

References

- [1] Inoue, T. (2006), "Indivisible commodities and decentralization of strong core allocations," Unpublished Manuscript, Kyoto University.
- [2] Inoue, T. (forthcoming), "Indivisible commodities and the nonemptiness of the weak core," *Journal of Mathematical Economics*.