Pensions and fertility: in search of a link

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Introduction

- Fertility and pensions: complex relationship:
  - PAYGO has been blamed for decline in fertility.
  - Conversely, decline in fertility $\Longrightarrow$ challenge for PAYGO systems.
- The argument is based on the hypothesis that fertility is endogenous and that it can be (perfectly) controlled.
- The often mentioned source of the externality is that
  - The return of PAYGO depends on average number of children.
  - Individuals take only private benefits (utility) into account.
- This would then suggest that under PAYGO pension schemes result in a suboptimal number of children.

- To counter this, many economists have advocated a policy of linking pension benefits, and/or contributions, to the number of children.
- I will examine this question and the corresponding policy recommendation in this talk.
- A related issue in the context of pensions that I will also discuss is the question of human capital.
  - It is not just the number of workers that matter.
  - Resources available for funding the pensions of the retired depend not only on the **number** of the workers, but also on their **ability**.

- Whether or not one differentiates between the quantity and the quality of children, in assessing the relationship of the two with pensions, as well as in formulating policy recommendations, one has to also address the implications of:
  - (i) **Heterogeneity** of parents in child-rearing, or in tastes, for children.
  - (ii) **Uncertainty** associated with the quality/quantity of children.
    - One can not fully control the future quality of children through education.
    - Infertility, premature death, misplanning, and multiple births imply that one can not fully control quantity either.

- **Heterogeneity** matters because it introduces an element of **redistribution** into the picture. In consequence:
  - One may want to redistribute towards parents who find themselves to be poor because it costs them too much to raise children.
  - There may exist a tradeoff between the two objectives of (a) **having the "right" quality/quantity of children** and (b) **redistribution**.
  - The question of the tradeoff is all the more pertinent when the parents' child-rearing ability is not publicly observable and we have an **adverse selection** problem.

- **Uncertainty** matters because it introduces an element of **insurance** into the picture. In consequence:
  - One may want to insure parents against the risks associated with ending up poor because of too many children. [In models wherein children are expected to support their parents in retirement, as in Sinn (2004), the risk is to have too few and/or less able children].
  - There may exist a tradeoff between the two objectives of (a) **having the "right" number/quality of children** and (b) **fully insuring parents**.
  - The question of the tradeoff is all the more pertinent when the parents' effort in doing the right thing is not publicly observable and we have a **moral hazard** problem.

- Ideally, one wants to have one single model that can address all these issues together.
- It is easier, however, to break these various issues down and study them separately.
- To this end, I proceed as follows:
  - I start with a short background.
  - I present a basic OG model with endogenous fertility that underlies most of the discussion.

- - Initially, I assume that fertility is deterministic. This highlights the essence of the argument for subsidizing children.
  - Next, I introduce heterogeneity and the associated adverse selection problem.
  - Then, I introduce uncertainty and the associated moral hazard problem.
- Finally, I present a short overview of the ramifications of a modeling strategy that explicitly differentiates between the number and the quality of children. The modeling recognizes that the quality is more likely to be stochastic than fertility per se.

I. Background

- I start my account with Samuelson (1975). Although:
  - The earlier paper by Dasgupta (1969).
  - The earlier static question of the "optimum size of population" from Wicksell to Meade (1955).
- Samuelson (1975): While considering fertility to be **exogenous**, he was interested in ranking the steady states that emerge under different fertility rates.
- He discussed three different models:

1. Solow-Swan growth models: $\Rightarrow$ As low a fertility rate as possible
   - One maximizes per capita consumption:

$$
\begin{aligned}
c(n) &\equiv \max_{k} \ \{c = f(k) - (n-1)\,k\} \\
\frac{dc(n)}{dn} &= \frac{\partial c}{\partial n} = -k^{*} < 0,
\end{aligned}
$$

   - A declining population would yield higher per capita consumption as people can live off "narrowing" of capital.
2. OG models à la Samuelson (1958): $\Rightarrow$ As high a fertility rate as possible.
   - The "biological rate of interest" being an alternative to the rate of return on savings.

3. OG models à la Diamond (1965): $\Rightarrow$ The "Serendipity Theorem".

$$u(n) \equiv \max_{k,d} \left\{ u(c,d) = u\left(f(k) - (n-1)\,k - \frac{d}{n}, d \right) \right\},$$

$\Rightarrow$ Choose n to mazimize $u(n)$.

- Deardorff (1976):
  - Cobb-Douglas preferences $\Rightarrow$ Serendipity Th. yields a minimum.
- Michel and Pestieau (1993):
  - Complementarity between first and second period consumption, and complementarity in production, ensure the validity of the Serendipity Theorem.
  - (i) Intergenerational transfer and (ii) Capital dilution effects.

II. Endogenous and deterministic fertility

- The underlying model for my discussion is Diamond's (1965) two-period OG model.
- I assume that parents' utility depends on the number of their children.
  - The altruism is thus "one-sided" and "partial".
  - One sided in that the children do not care for their parents.
    - $\Rightarrow$ Thus ignoring that parents may want to have children as insurance.
  - Partial in that the parent's utility does not depend on the children's utility or their consumption as in Becker and Barro (1988).
- I also ignore family as a mechanism for intergenerational transfers as in Cigno (1993).

- The criterion I use for welfare is per capita steady state utility.
  - Exogenous fertility:
    - Generations on transitional paths.
    - Average versus total utility.
  - Endogenous fertility:
    - Conceptual difficulties.
    - Dasgupta (1995): Not even the right vocabulary.
    - Recent papers by Golosov *et al.* (2007), Michel and Wigniolle (2007), and others.

- First-best problem in the steady state:

$$\max_{c,d,n,k} u(c) + v(d) + \varphi(n),$$

- Subject to the economy's "yearly" resource constraint:

$$f(k) \geqq c + an + \frac{d}{n} + (n-1)\,k.$$

- $\Rightarrow$ First-order conditions

$$\begin{aligned} \frac{v'(d)}{u'(c)} &= \frac{1}{n}, \\ f'(k) &= n - 1. \end{aligned}$$

- $\Rightarrow$ First-order condition for n:

$$\frac{\varphi'(n)}{u'(c)} = a + \left(k - \frac{d}{n^2}\right).$$

- Externalities in the last equation.
  - Intergenerational transfer effect $(-d/n^2)$: Higher fertility $\Rightarrow$ increases the number of future working people who support a retired person.
  - Capital dilution effect (k): Higher fertility $\Rightarrow$ increases the number of future working people and thus increases the required capital to maintain the capital labor ratio.

Decentralization

- Instruments:
  - A tax per child, t,
  - A lump-sum tax on, or contribution of, the young, T,
  - Pensions, P.
- Individual's problem:

$$\mathcal{L} = u(c) + v(d) + \varphi(n) + \alpha \left[w - c - \frac{d - P}{1 + r} - (a + t)n - T \right].$$

- To attain first-best, one should set

$$\begin{cases} t^* = k - \frac{d}{n^2} \ \ \text{(where } k, d, n, \text{ are set at their FB values.)} \\ \\ P^* \text{ and } T^* \text{ are set so that GR is attained.} \end{cases}$$

- The sign of t^*:
  - The budget constraint of the old is given by

$$d - P = (1 + r)\, kn.$$

  - Substituting for k from this into the equation for t^* $\Rightarrow$

$$t^* = \frac{[n - (1 + r)]\, d - nP}{n^2\, (1 + r)}.$$

– $\Rightarrow$

$$t^* \lesseqqgtr 0 \Leftrightarrow P \gtreqqless \left(1 - \frac{1+r}{n}\right) d.$$

– Thus if $P > 0$ and $1 + r \geqq n$, then $t^* < 0$.

– At the GR, $n = 1 + r \Rightarrow$ The condition is reduced to

$$t^* \lesseqqgtr 0 \Leftrightarrow P \gtreqqless 0.$$

– In LF, $P = 0$ so that if n happens to be equal to $1 + r$,

$$t^* = 0.$$

- In Samuelson's OG model, capital dilution effect does not exist $\Rightarrow t^*$ is definitely a subsidy.

- Observe that the individual's intertemporal budget constraint

$$w - c - \frac{d - P}{1 + r} - (a + t)n - T = 0,$$

can be written as

$$w - c - \frac{d - [P - t\,(1 + r)\,n]}{1 + r} - an - T = 0.$$

- This suggests an *alternative* implementation

$$\begin{cases} \widehat{t} = 0, \ \text{(no child supplement)} \\ \widehat{P} = P^* + \tau\,(1 + r)\,n; \ \text{where } \tau = -t^* = \frac{d}{n^2} - k \\ \widehat{T} = T^*. \end{cases}$$

- This establishes the case for (i) child supplements, **or** (ii) relating pensions to fertility.

III. Parents' heterogeneity and redistribution

- Same Diamond's OG model in the steady state.
- Two types of parents in each generation.
- Differential cost of raising a child, a_j: $0 < a_h < a_l$.
- $\begin{cases} \pi_l = \text{ Proportion of } l\text{-type parents} \\ \pi_h = \text{Proportion of } h\text{-type parents} = 1 - \pi_l \end{cases}$
- No correlation between a parents' type and the type(s) of his children.
- Number of children of a j-type parent $= n_j$.
- The population growth rate is

$$\overline{n} = \pi_l n_l + \pi_h n_h.$$

Utilitarian first best

- First-best allocations as the solution to

$$\mathcal{L} = \sum_{j=l}^{h} \pi_j \left[u\left(c_j\right) + v\left(d_j\right) + \varphi(n_j) \right] + \mu \left[f(k) - \overline{c} - \overline{an} - \frac{\overline{d}}{\overline{n}} - \left(\overline{n} - 1\right) k \right].$$

- First-order conditions $\Rightarrow$

$$c_h = c_l \equiv c,$$
$$d_h = d_l \equiv d,$$
$$\frac{v'(d)}{u'(c)} = \frac{1}{\overline{n}},$$
$$f'(k) = \overline{n} - 1.$$

- First-order condition for n_j, $j = l, h$,

$$\frac{\varphi'(n_j)}{u'(c)} = a_j + k - \frac{d}{\overline{n}^2}.$$

- A j-type person pays only a_j for each child.
- $\Rightarrow$ Externality in choosing n_j:

$$k - \frac{d}{\overline{n}^2}.$$

Decentralization

- Requires a tax per child plus contributions of the young and pensions of the old:

$$\begin{cases} t^{*} = k - \frac{d}{\overline{n}^{2}}, \quad \text{(where } k, d, n_j \text{ are set at FB values).} \\ \\ P_j^{*} \text{ and } T_j^{*} \text{ are set for (i) redistribution and (ii) GR is attained.} \end{cases}$$

- Or

$$\begin{cases} \widehat{t} = 0, \ \text{(no child supplement)} \\ \\ \widehat{P}_j = P_j^{*} + \tau\,(1+r)\,n_j; \ \text{where } \tau = -t^{*} = \frac{d}{\overline{n}^{2}} - k \\ \\ \widehat{T} = T^{*}. \end{cases}$$

- Same message as before.

Adverse selection

- When parents' child-rearing abilities are publicly observable, there is no conflict between externality-correction and redistribution.
- But such individual characteristics are seldom publicly observable.
- $\Rightarrow$ There is asymmetric information between parents and the policy makers.
- One may not be able to distinguish between those individuals who have a small number of children due to high costs, and those with low costs who try to free ride on the system.

- This creates a tension between the two objectives of externality correction and redistribution.
- Under this circumstance, linking pension benefits to fertility penalizes high-cost families (who have few children). This in turn may have an adverse redistributive impact.
- Consequently, the externality-correcting property of relating pension benefits to the number of children may have to be balanced against its adverse redistributive potential.
- $\Rightarrow$ A positive link between fertility and pension benefits is not always desirable.

Adverse selection and second-best policy

- Simplify the model by leaving out capital accumulation.
- $\Rightarrow$ Samuelson's original OG framework.
- All individuals start life with identical and exogenous income I.
- With a_j, the cost of raising children, being publicly unobservable,
- $\Rightarrow$ A tax-transfer policy which induces type revelation.

- In Samuelson's framework, if PAYGO is the optimal system, one wants the consumption of the retired to be funded solely by their pensions.
- $\Rightarrow$ Set $P_j = d_j$.
- Consider a direct revelation mechanism offering packages (T_l, P_l, n_l) and (T_h, P_h, n_h).
- Let

$$\begin{aligned} c_{jk} &= I - a_j n_k - T_k, \\ U_{jk} &= u(c_{jk}) + v(P_k) + \varphi(n_k). \end{aligned}$$

- The government's budget constraint is given by

$$\sum_{j=l}^{h} \pi_j \left(T_j - \frac{P_j}{\overline{n}} \right) = 0.$$

- The second-best problem is summarized by the Lagrangian

$$\mathcal{L} = \sum_j \pi_j \left[U_j + \mu \left(T_j - \frac{d_j}{\overline{n}} \right) \right] + \lambda_h (U_h - U_{hl}) + \lambda_l (U_l - U_{lh}).$$

- Possible distortionary regimes:
  - (i) $\lambda_h > 0$ and $\lambda_l = 0$,
  - (ii) $\lambda_l > 0$ and $\lambda_h = 0$.
- An important determining factor is which type spends more on children.
- $\Rightarrow$ This in turn depends on the elasticity of substitution between consumption and fertility.

- FOC for the individual types one **distributes away from**:

$$\frac{h'(n_j)}{u'(c_j)} = a_j - \frac{\overline{d}}{\overline{n}^2}.$$

- The second-term in the right-hand side is the familiar intergenerational transfer externality term.
- There is no capital dilution term because of the Samuelson's framework.
- FOC for the individual types one **distributes towards**, has an **additional incentive term**:

$$\frac{h'(n_k)}{u'(c_k)} = a_k - \frac{\overline{d}}{\overline{n}^2} + \text{ an incentive term.}$$

Decentralization through the tax schedule $T(n)$

- The individual types one distributes away from face a Pigouvian *marginal* subsidy on n:

$$T'(n_j) = -\frac{\overline{d}}{\overline{n}^2} < 0.$$

- The types one distributes towards face

$$T'(n_k) = -\frac{\overline{d}}{\overline{n}^2} + \text{an extra term.}$$

- $\Rightarrow$

$$T'(n_h) \neq T'(n_l).$$

- $T'(n_h)$ and $T'(n_l)$ can even be of different signs!

- $\Rightarrow$ Not the policy recommendation that came from the model without heterogeneity.

The sign of the extra term

- (i) If it is the l-types one distributes towards:
  - Want to prevent the h-type from mimicking the l-type.
  - How to make the l-type's allocation less attractive to the h-type?
  - Because $a_h < a_l \Rightarrow h$ tends to prefer, relative to l, larger n.
  - $\Rightarrow$ Distort n_l downward.
  - Incentive term > 0.
- There is a conflict between externality (requiring a subsidy to induce a higher value for n_l) and incentive (requiring a tax to induce a lower value for n_l).
- The sign of $T'(n_l)$ is ambiguous.

- (ii) If it is the h-types one distributes towards:
  - Want to prevent the l-type from mimicking the h-type.
  - How to make the h-type's allocation less attractive to the l-type?
  - Because $a_l > a_h \Rightarrow l$ tends to prefer, relative to h, smaller n.
  - $\Rightarrow$ Distort n_h upward.
  - Incentive term < 0.
- The externality and incentive terms are of the same sign.
- $\Rightarrow$ Definitely a marginal subsidy on n_h (a negative marginal tax).

IV. Endogenous and partly stochastic fertility/quality.

- Same two-period overlapping generations model in the steady state.
- A parent can have either n_l or n_h children, with $n_h > n_l$.
- Realization of n_j depends on "investment in children," e, and on some random shock.
  - Probability of $n_h : \pi_h(e) = \pi(e)$ where $\pi'(e) > 0$;
  - Probability of $n_l : \pi_l(e) = 1 - \pi(e)$.
- As before, there is also a cost of raising a child, a.
- Expected utility:

$$U = \sum_{j=l}^{h} \pi_j(e) \left[u(c_j) + v(d_j) + \varphi(n_j) \right].$$

- Define

$$\begin{aligned}\overline{n}(e) &= \pi_l(e)\, n_l + \pi_h(e)\, n_h, \\ \overline{c}(e) &= \pi_l(e)\, c_l + \pi_h(e)\, c_h, \\ \overline{d}(e) &= \pi_l(e)\, d_l + \pi_h(e)\, d_h.\end{aligned}$$

- Economy's resource constraint in the steady-state,

$$f(k) \geqq \overline{c}(e) + e + a\overline{n}(e) + \frac{\overline{d}(e)}{\overline{n}(e)} + [\overline{n}(e) - 1]\, k.$$

- First-order conditions

$$
\begin{aligned}
c_h &= c_l \equiv c, \\
d_h &= d_l \equiv d, \\
\frac{v'(d)}{u'(c)} &= \frac{1}{\overline{n}\,(e)}, \\
f'(k) &= \overline{n}\,(e) - 1.
\end{aligned}
$$

- First-order condition w.r.t. n:

$$\frac{[\varphi(n_h) - \varphi(n_l)]\,\pi'(e)}{u'(c)} = 1 + (n_h - n_l)\,\pi'(e)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right).$$

  – The numerator in the left-hand side shows the marginal benefit to the individual of spending one extra dollar on e.

  – The denominator in the left-hand side shows the marginal benefit to the individual of spending one extra dollar on c.

  – These are equal at the optimum.

  – Individual sets the left-hand side equal to one.

- $\Rightarrow$ The externality term is

$$(n_h - n_l)\,\pi'(e)\left(a + k - \frac{\overline{d}}{\overline{n}^2}\right).$$

- As an individual increases his investment in fertility, he will increase the probability of having $(n_h - n_l)$ *more* children by $\pi'(e)$.

- The extra cost imposed on the society is thus $\pi'(e) \times (n_h - n_l) \times \left(a + k - \overline{d}/\overline{n}^2\right)$.

- The terms k and $\overline{d}/\overline{n}^2$ are as previously.

- The extra term is a, the additional expected cost to the society in raising $(n_h - n_l)$ more children.

Decentralization

- Instruments: a tax on investment (in fertility or in education), t, plus contributions of the young and pensions of the old.

$$\begin{cases} t = (n_h - n_l)\,\pi'(e)\left(a + k - \frac{\bar{d}}{\bar{n}^2}\right), \\ \\ T_j \text{ and } P_j \text{ to (i) attain GR, (ii) equalize consumption.} \end{cases}$$

- True that the size of t varies with the difference between n_h and n_l; but t is not being given *per child*.

Moral hazard

- Stochastic fertility/education, added a dimension of insurance to the problem.
- As long as investment in fertility or in education, e, is publicly observable:
  - It is e which will be subsidized.
  - One attains both objectives of a "correct" level of e and full insurance.
- With public unobservability of e, a moral hazard problem arises in the choice of e which the government tries to correct by subsidizing n.
- This creates a tradeoff between achieving the correct value of e, through a subsidy on n, and achieving full insurance.

- It is again easier to work with the Samuelson's version of the OG model.
- All individuals start life with identical and exogenous income I.
- Continue to assume that n_j's $(j = l, h)$ are publicly observable.
- Set up a two-stage optimal tax problem **when the instruments do not include a tax on e.**
- Second stage: The young choose (c_l, d_l, n_l) and (c_h, d_h, n_h) to maximize expected utility when facing the policy instruments T_l, T_h, P_l and P_h.
- First stage: the government optimizes over the instruments (anticipating the individuals' optimization problem).

- The second-best allocation requires:
  - Pension benefits that increase with the number of children: $P_h > P_l$ so that $d_h > d_l$.
  - Contributions that decrease with the number of children: $T_h < T_l$.
  - There is a tradeoff between externality-correcting and insurance considerations $\Rightarrow$
    * A level of investment in children, and a resulting fertility rate, that are below their corresponding first-best levels.
    * Parents are more than compensated for the extra cost of children: $T_l - T_h > a(n_h - n_l)$ and $c_h > c_l$.

Model IV: Separating education from fertility

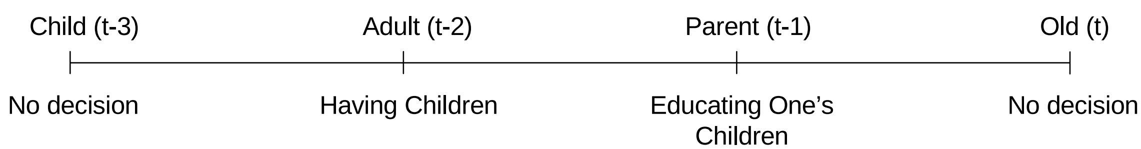

Figure 1:

- CGP (2008).
- Number of children, n, is deterministic.
- Children's ability is partly random and partly determined through education, e.

- An important new factor that comes into play here is the impact of a parent's choice of n and e on the proportion of high-ability individuals in the steady state:

$$\delta = \delta(n_l, n_h, e_l, e_h).$$

- This function has the following properties:

$$\frac{\partial \delta}{\partial e_h} > 0, \ \frac{\partial \delta}{\partial e_l} > 0; \quad \frac{\partial \delta}{\partial n_h} \text{ and } \frac{\partial \delta}{\partial n_l} \text{ are of opposite signs.}$$

- There is one source of externality in the choice of e. This is its impact on δ which is positive.

- To correct this externality we have to impose a subsidy, $-\tau_j$, on e_j.

- Three sources of externality associated with the choice of n:
  - The familiar negative capital dilution effect.
  - The familiar positive intergenerational transfer effect.
  - A third source of externality emanating from a change in δ.
    - Of opposite sign for n_h and n_l.
- This externality is corrected by a **net** tax per child equal to $(t_j + \tau_j e_j)$.
- Note that:
  - Each child is directly "taxed" by t.
  - Each child is indirectly "taxed" by τe.

- We have:
  - τ_j to be a subsidy.
  - $(t_j + \tau_j e_j)$ to be a subsidy.
  - It is not clear with t_j.

Conclusion

- In the presence of PAYGO pension schemes, the decentralized choice of fertility by parents is distorted.
- The distortion is typically downward resulting in a smaller than optimal number of children.
- There is a case for positively linking pensions to the number of children to correct this, provided that fertility can be perfectly controlled and that no parents are better than others in raising their children.
- With parents' heterogeneity, the government has redistributive aims as well.
  - However, public unobservability of parents' effort in raising children gives rise to an adverse selection problem.

        * ⇒ The externality-correcting property of relating pension benefits to the number of children will then have to be balanced against its adverse redistributive potential.
        * ⇒ A positive link between fertility and pension benefits is not always desirable.

- Similarly, when fertility is partly stochastic and cannot be fully controlled, the government may want to insure parents against the risks associated with having too many children.
    - If the parents' investment in fertility/education is publicly unobservable, a moral hazard problem arises.

    - $*$ $\Rightarrow$ This creates a tradeoff between achieving the correct value of e, through a subsidy on n, and achieving full insurance.
    - $*$ $\Rightarrow$ The balance of the tradeoff calls for too low an investment level and over-insuranceFinally, when fertility decision is considered separately from investment in education, subsidizing children can be achieved though educational subsidies and child supplements.
  - While the first element is necessarily positive, this may not be the case with the second element.